U.G. 3rd Semester Examination - 2020

MATHEMATICS

Course Code : BMTMCCHT303

Course Title : Geometry-3D and Vector Analysis

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Find the equation of the right circular cylinder whose axis is the x-axis and radius 1.

b) Find the direction cosines of the line which is equally inclined to the axes in three dimensions.

c) If $\vec{u} = 2\hat{i} - 2\hat{j} - \hat{k}$ and $\vec{v} = \hat{i} + \hat{j} + \hat{k}$, find $|\vec{u} \times \vec{v}|$.

d) Find the equation of the plane passing through P(a, b, c) and perpendicular to OP, where O is the origin.

e) Find the equation of the right circular cone whose vertex is origin, axis is the z-axis and semi-vertical angle $\frac{\pi}{4}$.

f) Find the coordinates of the point in which the straight line $\frac{x-1}{3} = \frac{y+2}{-1} = \frac{z}{4}$ intersects the plane $4x + y + z = 2$.

g) Show that the vector $\left\{(\vec{\alpha}.\vec{\gamma})\vec{\beta} - (\vec{\alpha}.\vec{\beta})\vec{\gamma}\right\}$ is perpendicular to the vector $\vec{\alpha}$.

h) Show that $\vec{a} \times (\vec{b} \times \vec{c})$ is coplanar with $\vec{a} \times \vec{b}$.

i) Find the unit vector perpendicular to each of the vectors $\vec{a} = \hat{i} - 2\hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$.

j) If $u = x^3 + 3yz^2$, then find $\nabla^2 u$.

k) Find the nature of the quadric surface given by the equation

$$2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z = 5.$$

l) State Stokes' theorem.

m) Evaluate

$$\int_1^2 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right) dt, \text{ where } \vec{r} = 2t^2\hat{i} + t\hat{j} - 3t^2\hat{k}.$$

n) Show that the curl of gradient is zero.

o) Find a unit normal to the surface $2x^2y+3yz=4$ at the point (1, –1, –2).

2. Answer any **five** questions: 2×5=10

a) Examine if the lines
$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$
are coplanar.

b) Find x so that the vectors $2\hat{i}+\hat{j}+3\hat{k}, 3\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+x\hat{k}$ may be coplanar.

c) Find the ratio in which the straight line joining the points (2,–3,5) and (7,1,3) is divided by the xy-plane.

d) A particle moves according to the law $\vec{r}=\cos t\,\hat{i}+\sin t\,\hat{j}+t^2\hat{k}$. Find the magnitudes of the tangential components of acceleration.

e) Find the equation of the cylinder whose generating line is parallel to the z-axis and the guiding curve is $x^2+y^2=z, x+y+z=1$.

f) Show that $\nabla^2\left(\frac{1}{r}\right)=0$ where $r=\sqrt{x^2+y^2+z^2}$.

g) Find the equation of the surface of revolution whose generatrix is $z^2=4y, x=0$ about the y-axis.

h) Find the equation of the cone whose vertex is the origin and base is the circle $x=a,\ y^2+z^2=b^2$.

3. Answer any **two** questions: 5×2=10

a) Show that the equation of the plane containing the straight line $\frac{y}{b}+\frac{z}{c}=1, x=0$ and parallel to the straight line $\frac{x}{a}-\frac{z}{c}=1, y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$.

b) Verify Green's theorem in the plane for $\int_c \left(x^2-xy^2\right)dx+\left(y^2-2xy\right)dy$ where c is the square with vertices (0,0), (2,0), (2,2) and (0,2).

c) Find the equations of the lines in which the plane $2x-6y-5z=0$ cuts the cone $xy+yz+zx=0$.

4. Answer any **one** question: 10×1=10

a) i) Find the locus of a luminous point, if the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ casts a circular shadow on the plane z=0.

ii) Evaluate $\oint_\Gamma \vec{F}.d\vec{r}$ by Stokes' theorem, where $\vec{F}=y^2\hat{i}+x^2\hat{j}-(x+z)\hat{k}$ and Γ is the boundary of the triangle with vertices at (0,0,0), (1,0,0), (1,1,0). 6+4

b) i) Reduce the equation :

$$6y^2-18yz-6zx+2xy-9x+5y-5z+2=0$$

to the canonical form and state the type of the quadric represented by it.

ii) Use divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,dS$, where $\vec{F}=3xz\hat{i}+y^2\hat{j}-3yz\hat{k}$ and S is the surface of the cube bounded by x = 0, y = 0, z = 0, x = 1, y = 1, z = 1. 6+4

c) i) A variable plane has intercept on the co-ordinate axes, the sum of whose square is k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(x^2+y^2+z^2\right)^2\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)=k^2.$$

ii) Prove that

$$\vec{F}=\left(y^2\cos x+z^3\right)\hat{i}+\left(2y\sin x-4\right)\hat{j}+\left(3xz^2+2\right)\hat{k}$$

is a conservative force field. Find the scalar potential V such that $\vec{F}=\vec{\nabla}V$. 5+5

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