U.G. 4th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMCCHT 401

Course Title : Dynamics of Particles

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) A particle describes the curve $r = ae^{\theta}$ with constant angular velocity; show that its radial acceleration is zero.

b) If the displacement of a moving point at any time be given by an equation of the form $x = a\cos kt + b\sin kt$, then show that the point executes a S.H.M.

c) Define kinetic energy. Write down its expression.

d) Define terminal velocity.

e) What is impulsive force and how is it measured?

f) From what height must a heavy elastic ball be dropped on a floor so that after rebounding once it will reach a height of 4 metres? $\left(\text{Given } e = \frac{2}{3}\right)$.

g) The position of a moving particle at time t is given by $x = a\cos nt$, $y = a\sin nt$. Find its acceleration.

h) A parrticle describes a curve $r = ae^{\theta}$ with constant angular velocity. Show that its transverse acceleration varies as the distance from the pole.

i) Write down the relation between linear velocity and angular velocity of a particle moving in a plane.

j) If a particle moves in a circle of radius r with a uniform speed v, then its angular velocity about the centre is uniform and is equal to $\frac{v}{r}$, prove it.

k) Prove that at an apse, p=r, where the symbols have their usual meaning.

l) State Newton's law of gravitation.

m) If the tangential and normal acceleration of a particle moving in a plane curve are equal, find the expression for the velocity.

n) If a particle be projected with a velocity u at an angle 45^{o} to the horizontal, then what will be the maximum range of the particle?

o) State the condition of stability for a circular orbit under central force μu^{n} with centre of force at the centre of the circle.

2. Answer any **five** questions: $2\times5=10$

a) Prove that for a particle moving with uniform acceleration f in a straight line

$$f = 2.\frac{\frac{s'}{t'} - \frac{s}{t}}{t + t'},$$

where s is the space described in t seconds and s′ during next t′ seconds.

b) Show that in a S.H.M. of amplitude a and period T the velocity v at a distance x from the centre is given by the relation

$$v^2T^2 = 4\pi^2\left(a^2 - x^2\right).$$

c) A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoils with a velocity $\sqrt{\frac{2mE}{M(M+m)}}$.

d) A particle moving in a straight line according to the law of ax^2+bx+c where x is the distance from a fixed point on the line of motion and a, b, c are constants. If v be the velocity in time t, show that the retardation of the particle is $2av^3$.

e) An insect crawls at a constant rate u along the spoke of a cart wheel of radius a, the cart moving with a constant velocity v. Find the acceleration along and perpendicular to the spoke.

f) Show that a central orbit is always a plane curve.

g) Find the period of a small oscillation of a simple pendulum.

h) Two smooth spheres of masses m_1 and m_2 moving with respective velocities u_1 and u_2 in the same direction impinge directly. If e be the coefficient of restitution between them, find their velocities after impact.

3. Answer any **two** questions: 5×2=10

a) A particle moves with an acceleration which is always towards, and equal to μ divided by the distance from a fixed point O. It is starts from rest at a distance 'a' from O, show that it will arrive at O in time $a\sqrt{\frac{\pi}{2\mu}}$.

b) A particle of mass m moves under a central attractive force $m\mu\left(5u^3+8c^2u^5\right)$ and is projected from an apse at a distance c with velocity $\frac{3\sqrt{\mu}}{c}$. Prove that the orbit is $r=c\cos\frac{2}{3}\theta$.

c) A particle is projected vertically upwards with a velocity u in a medium whose resistance varies as the square of the velocity. Show that the particle comes to rest at a height $\frac{V^2}{2g}\log\left(1+\frac{u^2}{V^2}\right)$, where V is the terminal velocity.

4. Answer any **one** question: 10×1=10

a) i) A particle is projected vertically upwards with a velocity u and the resistance of air produces a retardation kv^2, where v is the velocity and k is a constant. Show that the velocity u_1 with which the particle will return to the point of projection is given by

$$\frac{1}{u_1^2}=\frac{1}{u^2}+\frac{k}{g}.$$

ii) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil of the gun is $\left\{\frac{2m^2gh}{M(m+M)}\right\}^{\frac{1}{2}}$. 6+4

b) i) A particle is projected with a velocity u at an angle α to the horizon in a medium whose resistance is mk times the velocity. Find the equation of the trajectory. 6

ii) State Kepler's third law of planetary motion. Establish the modified law. 4

c) i) A heavy particle slides down a rough cycloid of which the coefficient of friction is μ. Its base is horizontal and vertex downwards. Show that if it starts from rest at the cusp and comes to rest at the vertex, then $\mu^2 e^{\mu\pi} = 1$. 6

ii) A particle moves with a central acceleration $\frac{\mu}{r^2} - \frac{\lambda}{r^3}$, where r is the distance from the centre and λ, μ are constants. Show that the apsidal angle is $\pi \div \sqrt{1 + \frac{\lambda}{h^2}}$, where $\frac{h}{2}$ is the constant areal velocity. 4

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