U.G. 4th Semester Examination - 2021

MATHEMATICS

Course Code : BMTMCCHT 402

Course Title : Partial Differential Equation, Laplace Transform and Tensor Analysis

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Define linear partial differential equation (P.D.E.).

b) Write down the general form of quasi-linear P.D.E.

c) Let $f(x, y, z, p, q)=0$ be a P.D.E., then write down the auxiliary equation for Charpit's Method.

d) Give an example of a first degree P.D.E.

e) Give the geometrical interpretation of the Lagrange's equation $P\frac{\partial z}{\partial x}+Q\frac{\partial z}{\partial y}=R$, where P, Q, R are functions of x, y, z.

f) Write down the value of $L\{e^{kt}\}$.

g) Find $L\{(t^2+1)^2\}$.

h) If $L\{F(t)\}=f(p)$, then prove that $L\{F(at)\}=\frac{1}{a}f\left(\frac{p}{a}\right)$.

i) Write down the value of $L^{-1}\left\{\frac{p}{p^2+k^2}\right\}$.

j) State the convolution theorem.

k) Define invariant or scalar.

l) Define anti symmetric tensor.

m) Prove that $\begin{Bmatrix} p \\ ij \end{Bmatrix}=\begin{Bmatrix} p \\ ji \end{Bmatrix}$.

n) State Ricii theorem.

o) Define curl of a vector.

2. Answer any **five** questions: 2×5=10

a) Eliminate the arbitrary constants from $z=(x-a)^2+(y-b)^2$ to form the P.D.E.

b) Solve the P.D.E., p+q=1.

c) Write down the general form of semi-linear P.D.E.

d) State the sufficient condition for the existence of Laplace Transform.

e) Find $L\{f(t)\}$, where $f(t)=\begin{cases} e^t, & 0<t\le 1 \\ t, & t>1 \end{cases}$.

f) Find the value of $L^{-1}\left\{\dfrac{4}{p-3}+\dfrac{p+6}{p^2+4}\right\}$.

g) Prove that $\delta_i^i = n$.

h) Prove that Kronecker delta δ_j^i is a mixed tensor of rank 2.

3. Answer any **two** questions: 5×2=10

a) Find the complete and singular solution of the P.D.E., $2xz-px^2-2qxy+pq=0$.

b) Using Laplace transform, solve the I.V.P. $y''(t)+2y'(t)+y(t)=3te^{-t}$, given that y(0)=4, $y'(0)=2$.

c) Prove that $R_{ijk,m}+R_{ikm,j}+R_{imj,k}=0$.

4. Answer any **one** question: 10×1=10

a) i) Solve the P.D.E.

$(mz-ny)p+(nx-lz)q+(nx-lz)q=ly-mx$.

ii) The components of a contravariant tensor in the co-ordinate system x^i are $A^{11}=4$, $A^{12}=A^{21}=0$, $A^{22}=7$. Find its components in co-ordinate system $\bar{x}^i$, where $\bar{x}^1=4(x^1)^2-7(x^2)^2$, $\bar{x}^2=4x^1-5x^2$.

5+5=10

b) i) Find the complete integral of $z^2(p^2z^2+q^2)=1$.

ii) Find the Laplace transform of

$$f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$$

where $f(t+2a)=f(t)$. 5+5=10

c) i) Use convolution theorem to solve the I.V.P. $y''(t)+y(t)=\sin 3t$, given that $y(0)=0=y'(0)$.

ii) If A^{ijk} is a skew symmetric tensor, show that $\dfrac{1}{\sqrt{g}}\dfrac{\partial}{\partial x^k}\left(\sqrt{g}\,A^{ijk}\right)$ is a tensor.

5+5=10
