U.G. 5th Semester Examination - 2021

MATHEMATICS

Course Code: BMTMDSHT2 [DSE 2]

Course Title: Mechanics-1

Full Marks : 40   Time : 2 Hours

The figures in the right-hand margin indicate marks.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: $1 \times 10 = 10$

    a) Give an example of non-inertial frame of reference.

    b) State the theorem of parallel axis.

    c) Give an example of a motion in which both body force and surface are present.

    d) Write down the work done by a force in a moving particle round a closed curve.

    e) What is the dimension of viscosity?

    f) What is the principle of conservation of energy?

    g) What is the difference between ideal fluid and viscous fluid?

    h) Is cylindrical co-ordinate system an inertial frame? Justify.

    i) What do you men by 'Motion in two Dimension'?

    j) Does kinetic energy of a particle remain invariant under Gallilean transformaion? Justify your answer.

    k) Obtain product of inertia of an elliptic lamina with respect to its major and minor axes.

    l) Show that the distance between two points is invariant under Gallilean transformation.

    m) Two particles move under their mutual gravitational attraction. Write down their equations of motion.

    n) Give an example of each - body force and contact force.

    o) Is it correct that D'Alembert's principle as conjunction of the principle of virtual work and Newton's 2nd law of motion? Justify your answer.

2. Answer any **five** questions: 2×5=10

a) Deduce the expression for the angular momentum of a rigid body relative to the centre of mass.

b) Find the inertia matrix of a rectangular plane bounded by x = 0, x = 2a, y = 0, y = 2b, z = 0 referred to coordinate axes.

c) Establish 'Newton's first law of motion is a special case of Newton's second law of motion'.

d) What is simple pendulum? Write down its kinetics energy.

e) Establish the relation between the rate of change of angular momentum of a moving particle and the force acting on it.

f) Give the physical significance of the eigen value's and eigen vectors of the inertia matrix.

g) If the position of the fixed axis is change, then explain briefly wheher there is any change occurs in the expression for kinetic energy of a rigid body moving about the fixed axis.

h) Distinguish between sliding and rolling of a rigid body on a rough surface.

3. Answer any **two** questions: 5×2=10

a) A uniform solid cylinder of mass M and radius a is rolled with its axis horizontal up a rough inclined plane by means of constant couple L. Show that for this to be possible, the coefficient of friction must exceed $\frac{1}{3}\tan\alpha \frac{2}{3}\frac{L\sec\alpha}{Mga}$, the plane inclined at an angle α to the horizon.

b) A uniform circular disc is projected up a rough inclined plane with velocity V, but no rotation, the plane of the disc being a plane of greatest slope. Show that the disc ceases to slip after a time $\frac{V}{g(3\mu\cos\alpha + \sin\alpha)}$; μ and ∞ are respectively the co-efficient of friction and inclination of the plane.

c) A uniform solid sphere of radius a rolls down an inclined plane rough enough to prevent sliding. Write the equations of the sphere. Find the acceleration of the centre down the plane and show that for pure rolling $\mu > \frac{2}{7}\tan\alpha$; α being the inclination of the plane to the horizontal and μ, the co-efficient of function.

4. Answer any **one** question: 10×1=10

a) i) Find the condition that a given straight line at any point of its lenght is a principal axis of a given body and if so, find the other two principal axes.

ii) Prove that the momental ellipsoid at the centre of the elliptic plate is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \left(\frac{1}{a^2} + \frac{1}{b^2}\right)z^2 = \text{constant.} \quad 8+2$$

b) i) 'If a frame of reference is moving with constant speed relative to an inertial frame of reference; then it is also an inertial frame of reference; Is the statement correct? Justify your answer with reason.

ii) A system of n particles moving under external forces and their mutual actions and reactions. Show that the internal forces form a system in equilibrium and also show that the centre of mass of the system moves as if the whole mass is concentrated at the centre of mass and the external forces are acting at that point. 3+7

c) i) Show that the moment of momentum about a fixed point O of a rigid body of mass M moving in two dimension is equal to $Mvp + Mk^2\dot{\theta}$; explain the notations involved.

ii) A uniform rod is held at an inclination α to the horizon with one end in contact with a horizontal table whose co-efficient of friction is μ. It is then released. Write its equations of motion at any time t and show that it will commence to slide if $\mu < \dfrac{3\sin\alpha\cos\alpha}{1+3\sin^2\alpha}$. 5+5

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